Dielectric Spectra Reconstruction of Layered Multi-Phase Soil

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Abstract—A broadband soil dielectric spectra retrieval approach (1 MHz to 2 GHz) has been implemented for a layered half space. The inversion kernel consists of a two port transmission line forward model in the frequency domain and a constitutive material equation based of a power law soil mixture rule (Complex Refractive Index Model - CRIM) considering (i) the volume fractions of the soil phases, (ii) dielectric relaxation of the aqueous pore solution, (iii) electrical losses and (iv) low frequency dispersion due to the interactions between the pore solution and solid particles. The spatially distributed reconstruction of broadband dielectric spectra is achieved with a global optimization approach based on a Shuffled Complex Evolution (SCE) algorithm using the full set of the scattering parameter. The possibilities and limitations of the inverse parameter estimation were numerically analyzed.

Index Terms—Dielectric measurements, Soil measurements, Electromagnetic scattering inverse problems

I. INTRODUCTION

A quantitative temporal and spatial retrieval of broadband electromagnetic (EM) material properties is needed in various fields of applications e.g. agriculture, civil engineering, environmental research and hydrology.

The raw dielectric spectra of layered soil samples can be used to e.g. validate and calibrate remote sensing data.

But an EM signal with bandwidth from 1 MHz to 2 GHz contains more valuable information of a measured porous material due to the contribution to the dielectric relaxation behavior by the [15], [19], [12], [17], [1]: Bulk HF-EM properties of the volume fractions (solid particles, air, aqueous pore solution), Geometrical properties of the bulk phases (particle shape, pore size distribution), Mobility of charges in the pore network (extra and intra-aggregate porosity) from nm to m and Interactions between aqueous pore solution and mineral phases due to interface effects.

This opens the possibility to estimate physicochemical parameters from a broadband dielectric spectrum of the material beside volumetric water content such as porosity, texture, mineralogy and hydraulic properties with broadband HF-EM measurement techniques.

However, there is a lack of studies or approaches for the spatial estimation of broadband dielectric spectra. The existing methods suffer from high computational costs, like in full 3D numerical modeling [20] or they use Debye relaxation on a regular grid [9], [16], [10], [14], [11].

The frequency domain transmission line measurement, modeling and inversion with non-Debye relaxation for multilayer setups offers a good compromise of broad frequency bands, low computation costs, high complexity of materials mixtures and layer setups.

II. 1D TRANSMISSION LINE MODEL WITH MULTI-PHASE MATERIAL

The propagation of an EM wave along a transmission line surrounded by a certain material can be characterized by a scattering matrix \( S = (S_{ij}) \). The four complex scattering parameters \( S_{ij} \) with \( i,j \in \{1; 2\} \) are modeled based on the frequency-temperature-pressure dependent complex effective permittivity \( \varepsilon_{eff} \) of the material and layer thickness following [8] using EM propagation matrices.

The material properties of one layer \( k \in \mathbb{N} \) are considered by the permittivity dependency of the propagation constant \( \gamma_k \) and the characteristic impedance \( Z_k \).

In general the complex relative effective dielectric permittivity \( \varepsilon_{r, eff} \) of a multi-phase material containing an aqueous solution depends on temperature, pressure and frequency. The dispersive behaviour of a moist soil can be described with
the theoretical power law model with exponent equal to 1/2 (CRIM): [17], [18], [4]

$$\sqrt{\epsilon_{r,eff}^s(S_W, \omega, n)} = S_W n \sqrt{\epsilon_{r,W}^s(\omega)+(1-n)} \sqrt{\epsilon_{r,G}^s+n(1-S_W)}. \quad (1)$$

Here, $\epsilon_{r,W}^s$ is the complex relative permittivity of the aqueous pore solution, $S_W$ the water saturation, $n$ the porosity and $\epsilon_{r,G}^s$ the complex relative permittivity of the mineral matrix, e.g. solid particles. This formula is a weighted sum of the properties related to the travel time of the EM wave through the three materials: aqueous pore solution, the solid particles (e.g. quartz, feldspar, mica, kaolinite, vermiculite [13]) and air. It is only valid if the mixture is homogeneous and isotropic at the sample scale which is itself related to the applied wavelength.

The pore water is assumed to be the only frequency dependent material in this mixture and is represented by its conductivity $\sigma_W$ in the considered frequency range from 1 MHz to 10 GHz [19], [18], [4].

$$\epsilon_{r,W}^s = \epsilon_{\infty} + \frac{\epsilon_s - \epsilon_{\infty}}{1 + i \omega \tau}. \quad (2)$$

Here, $\epsilon_{\infty}$ is the high-frequency permittivity, $\epsilon_s$ the static permittivity and $\tau$ the dielectric relaxation time, with $\tau = (2\pi f_{rel})^{-1}$, where $f_{rel}$ is the relaxation frequency. Furthermore, pore water shows a considerable direct current electrical conductivity $\sigma_W$, which contributes to the losses [19]:

$$\epsilon_{r,W}^s = \epsilon_{r,D}^s(\omega) + i \frac{\sigma_W}{\omega \epsilon_0} = \epsilon_{r,D}^s - i \left( \epsilon_{r,D}'' + \frac{\sigma_W}{\omega \epsilon_0} \right). \quad (3)$$

The used broadband three-phase CRIM model includes water relaxation, interfacial relaxation as well as losses due to direct-current conductivity. From a practical point of view, the supposed model is sufficient for most soils and rocks in the considered frequency range from 1 MHz to 10 GHz [19], [18], [4].

Every expected layer of the sample is modeled with (1) depending on its material properties. With the complex relative effective dielectric permittivity $\gamma_k$, $Z_k$ and finally $S_k$ of the layer are calculated. By chaining these matrices the overall scattering parameter $S_{model}$ are gained. For a three layer setup this results in a set of 11 parameters to estimate.

### III. INVERSE PARAMETER ESTIMATION

We use a SCE optimization algorithm based on an analytical forward model of the scattering parameters $S_{model}$ to fit the synthetically generated and noisy measurement $S_{noise}$. To minimize the distance between the noisy synthetic data and modeled scattering parameters $S_{ij}$, we employed the objective function

$$F(p) = \sum_{i,j=1}^{2} \sum_{k=1}^{Q} \left\| S_{model}^{ij}(p, \omega_k) - S_{noise}^{ij}(\omega_k) \right\|^2 \Rightarrow \min$$

Here, $p$ is the parameter set, $Q$ the number of frequency points $\omega_k$, and $\|\cdot\|$ the complex absolute value function.

For the variation of the parameters the SCE algorithm introduced by [5] with parameter adjustments of [2] is used. When an optimal parameter set is found the likely range of the optimized parameter has to be determined. Here we use the Monte Carlo Markov Chain (MCMC) methods [7] to estimate the parameter distributions. These calculations furthermore deliver useful informations about the model quality. Especially the correlations between parameters can be investigated.

### IV. EXPERIMENTAL SETUP

To study the performance and precision of the algorithm in case of dielectric dispersion and electrical losses, synthetic data corresponding to the setup presented in Table I is generated with the previously introduced forward model. Here, the broadband dielectric spectra including the corresponding parameter porosity, water saturation and electrical conductivity of the aqueous pore solution are reconstructed simultaneously for each layer to analyze the method with environmental materials analogous to soil.

This hydrogeological relevant setup describes the temporal hydraulic sealing of a top sandy layer by sedimentation of silt and clay fractions in the pore space. In such a case porosity $n_k$ is reduced during the sedimentation and the top layer of reduced $n_k$ becomes thicker. To model $\epsilon_{r,eff}$ of each layer, the CRIM formula (1) is used where the Debye parameters of water ($\epsilon_{\infty}$, $\epsilon_s$, $\tau$) are fixed and taken from [6] at a temperature $T = 20.6 ^{\circ}C$. The used permittivity of the solid particles four all setups are set to $\epsilon_{r,G}^s = 5$ and $\epsilon_{r,G}'' = 0$ which corresponds to a typical organic free soil.

Moreover, Gaussian noise was added to the modeled $\epsilon_{r,eff}(\omega)$ of each layer. With a variance $\sigma = 0.1$ of the normal distribution a real measurement is simulated. The noise is applied to the real- and imaginary part individually.

### V. RESULTS

In Figure 1 an example of a forward modeled broadband spectrum including the noise is plotted together with the retrieved spectra obtained from inverted material parameters. It is the spectrum of the first layer $k = 1$. It is a relatively thin layer with low water content. The retrieved and the synthetically generated spectra show a deviation (see Table II). The mismatch of the spectra of layer $k = 1$ is stronger than the one of layer $k = 2$. The reason here is the lower influence of the layer parameters on the overall result. Especially the water content plays an important role: it mainly determines...
TABLE II

<table>
<thead>
<tr>
<th>Layer</th>
<th>Parameter [unit]</th>
<th>Model</th>
<th>Fit $\mu$</th>
<th>Uncert. $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1$</td>
<td>$n_1$ [-]</td>
<td>$2 \times 10^{-1}$</td>
<td>$1.1 \times 10^{-1}$</td>
<td>$1.12 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{w,1}$ [S m$^{-1}$]</td>
<td>$3 \times 10^{-1}$</td>
<td>$3.5 \times 10^{-1}$</td>
<td>$3.95 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>$d_1$ [m]</td>
<td>$1 \times 10^{-1}$</td>
<td>$1 \times 10^{-1}$</td>
<td>$3.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$k = 2$</td>
<td>$n_2$ [-]</td>
<td>$4.5 \times 10^{-1}$</td>
<td>$4.7 \times 10^{-1}$</td>
<td>$1 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{w,2}$ [S m$^{-1}$]</td>
<td>$5 \times 10^{-2}$</td>
<td>$5.4 \times 10^{-2}$</td>
<td>$9.6 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$d_2$ [m]</td>
<td>$9 \times 10^{-1}$</td>
<td>$9 \times 10^{-1}$</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>$k = 3$</td>
<td>$n_3$ [-]</td>
<td>$4.5 \times 10^{-1}$</td>
<td>$4.5 \times 10^{-1}$</td>
<td>$1.81 \times 10^{-2}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{w,3}$ [S m$^{-1}$]</td>
<td>$3 \times 10^{-1}$</td>
<td>$3 \times 10^{-1}$</td>
<td>$2.5 \times 10^{-3}$</td>
</tr>
<tr>
<td></td>
<td>$d_3$ [m]</td>
<td>$1$</td>
<td>Fixed</td>
<td>—</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

The reasonable agreement of expected and optimized soil properties in the presented synthetic experiment shows that the developed model is suitable for a retrieval kernel. Moreover, the proposed frequency domain transmission line approach allows an automatic adaptation of layer number and thickness.

The used constitutive electromagnetic soil model enables the realistic consideration of dispersion as well as electrical and dielectric losses and thus the improved determination of spatial distributed dielectric relaxation behavior of the soil. Moreover, the proposed material model allows the coupling to soil hydraulic, mechanical and chemical soil properties [3].

We are currently working on the experimental validation of the proposed approach. Here we already tested layered structures of PTFE(Polytetrafluorethylen) and air in a coaxial cell waveguide, measured with a VNA(vector network analyser). The test of real soil samples will be implemented next.

REFERENCES


the losses, which furthermore causes the reduction of the transmission magnitude at higher frequencies.

For layer $k = 2$ and $k = 3$ the fitted parameters (see Table II) are in reasonable agreement with the model parameters and the uncertainty ranges are lower 1% of the fitted values. Only the one of the direct current conductivity $\sigma_{w,2}$ of layer $k = 2$ is about 13%. But the water content and the conductivity of this layer is relatively low and results in low imaginary parts $\epsilon''_{r,eff,2}(f)$. But even if the uncertainty range is added to and subtracted from the fitted value the conductivity stays in the typical dry loamy sand range and can clearly be distinguished from the higher conductivity of layer $k = 3$. The fitted parameters of layer $k = 1$ do not match that precisely. Only the value of the thickness $d_1$ is in good agreement to the model parameter. The other values show considerable mismatch. Especially the saturation $S_{w,1}$ has a difference of one order of magnitude. The parameters of such thin layers with low losses and low contrast to the adjacent layers have low influence on the objective function and thus can not be estimated precisely. But the contrast of porosity between the two layers is clearly visible and the clogging of a sand layer with former high hydraulic conductivity by sedimentation can be detected here.